sponse thermocouple and also indicate the same peak-to-peak temperature changes, thereby demonstrating acceptable response to small temperature changes.

VIII. Engine Tests

The TIGT system shown in Fig. 11 was installed on a J58 engine for flight suitability tests. Preliminary checks were made with a thermocouple installed in place of the fluidic sensor. The data taken during this test indicated that TIGT varied 67°C at the TIGT sampling location when EGT was constant. Therefore, it was anticipated that when on TIGT control there would be some EGT wander. Figure 12 is a plot of the system performance while on TIGT control. The range of EGT as shown in Fig. 13 was 30°C which is about ½ the earlier observed drift. This can be explained to some degree in that the EGT thermocouples have about a 2 sec time constant and the millivolt output of the interface selector also has a lag equal to about 2 sec. These two combine with the slow trim rate of the EGT vernier control to "clip" peaks that occur in the observed TIGT.

A post-test calibration was conducted on the system after 75 hr engine test time. The results of the sensor calibration are shown in Fig. 13. The postengine test calibration is slightly higher in frequency than the preengine test results. This is the direction of deterioration for a fluidic temperature sensor.

The hot gas sampling probe had accumulated 106 hr at the end of the engine test. A visual inspection of the probe revealed that the silicide coating was oxidized and flaky in places but there was no indication of base metal deterioration.

IX. Conclusions

1) The fluidic temperature sensor designed to measure the turbine inlet gas temperature of the J58 engine has been developed and qualified for flight tests in a YF-12 aircraft.

2) The transient response and steady-state accuracy of the TIGT measurement system meet the YF-12 flight test requirements.

3) Because of TIGT fluctuations at a single station, over-all engine control system performance cannot be adequately evaluated without a multiple TIGT sampling system.

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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Unsteady Vortex Flow Past an Inflating, Decelerating Wedge

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Nomenclature

b = half breadth of the wedge
C_p = pressure coefficient
K = constant defined by Eq. (4)
L = length of the rigging lines
n = α/π
p = pressure
q = velocity magnitude
s = distance measured along the surface
t = time
U = reference velocity
W = $\phi + i\psi$
x,y = coordinate axes
Z = $x + iy$
α = half apex angle of wedge
Γ = vortex strength
δ = slope of the rigging lines with the symmetry axis of the wedge
ζ = $\xi + i\eta$ an independent complex variable related analytically to W
ξ,η = coordinate axes
θ = direction of velocity vector
ρ = density
σ = $\log U/q + i\Theta$
ϕ = velocity potential
ψ = stream-function

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Index category: Nonsteady Aerodynamics.

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Introduction

THE inflation of a parachute from the line stretch condition to the fully inflated configuration is a complicated problem. Extensive studies have been made by Heinrich, Melzig, French, and others. However, their methods are primarily based on filling-time approach, where conveniently chosen inflow and outflow parameters completely determine the opening rates. More recently Roberts,[1] Wolf,[2] and Reddy[3] emphasized the importance of the aerodynamics of the unsteady flow in the canopy area.

In this Note, the parachute is approximated as a two-dimensional wedge as shown in Fig. 1. The problem of unsteady flow of an ideal fluid past an inflating, decelerating wedge is considered. The particular flow model shown in Fig. 2 is essentially one containing a wake flow with vortex sheet buried in and coinciding with the physical location of the canopy. As a first step an integral equation is developed which connects the rate of opening with the strength of the vortex sheet. Then, a second-order differential equation in time is obtained by equating the moment about the apex due to aerodynamic pressure acting

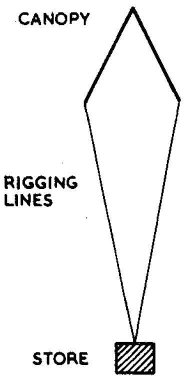

Fig. 1 Decelerator system and structural model employed.

across the canopy, and a further moment term due to the rigging line tension. Solution of this initial value problem gives the time history of the inflation of the wedge.

Unsteady Pressure Distribution on the Wedge Surface

In the Z-plane of Fig. 2 is shown a wedge with wetted surface ABB′ and wake CBB′C′. The relation between Z-, W-, σ-, and ζ-planes using the Schwarz-Christoffel formula is as follows:

$$W = K\zeta^2/2 \tag{1}$$

$$\sigma = \log\left[\frac{1}{\zeta} + \frac{1}{\zeta^2} - 1\right]^{2-2n} \tag{2}$$

$$Z = \frac{K}{U}\int_0^\zeta \frac{(1+[1-\zeta^2]^{1/2})^{2-2n}}{\zeta^{1-2n}}d\zeta \tag{3}$$

The breadth of the plate b is related to K as

$$b = \frac{K}{U}\int_0^1 \frac{(1+[1-\zeta^2]^{1/2})^{2-2n}}{\zeta^{1-2n}}d\zeta \tag{4}$$

It was stated in the Introduction that there is a vortex sheet placed to coincide with the physical location of the canopy. The strength of the vortex sheet is determined by equating the normal component of velocity due to the vortex sheet to the rate of opening of the wedge. This condition gives the following integral equation

$$\frac{1}{2\pi}\int_{-1}^{+1}\frac{\Gamma(\mu)}{\xi-\mu}d\mu = \left(\frac{K}{U}\right)^2\frac{d\alpha}{dt}\,\frac{(1+[1-\xi^2]^{1/2})^{2-2n}}{\xi^{1-2n}}\int_0^\xi \frac{(1+[1-\mu^2]^{1/2})^{2-2n}}{\mu^{1-2n}}d\mu \tag{5}$$

Equation (5) can be written in the form

$$\frac{1}{\pi}\int_0^1 \frac{\Gamma(\mu)d\mu}{\xi-\mu} = f(\xi) \tag{6}$$

where

$$f(\xi) = \left(\frac{K}{U}\right)^2\frac{d\alpha}{dt}\xi^{n-1/2}(1+[1-\xi]^{1/2})^{2-2n}\int_0^\xi \frac{(1+[1-\mu]^{1/2})^{2-2n}}{\mu^{1-n}}d\mu \tag{7}$$

The general solution of the integral Eq. (6) which is bounded at $\xi = 0$ is[4]

$$\Gamma(\xi) = \frac{1}{\pi}\left(\frac{K}{U}\right)^2\frac{d\alpha}{dt}\left(\frac{\xi}{1-\xi}\right)^{1/2}\int_0^1 \frac{F(\mu)d\mu}{\mu(\mu-\xi)} \tag{8}$$

where

$$F(\xi) = \xi^n[1-\xi]^{1/2}(1+[1-\xi]^{1/2})^{2-2n}\int_0^\xi \frac{(1+[1-\mu]^{1/2})^{2-2n}}{\mu^{1-n}}d\mu \tag{9}$$

Approximating for function $F(\xi)$ as a polynomial[5]

$$F(\xi) = \sum_{m=0}^{N} a_m \xi^m \tag{10}$$

the Eq. (8) reduces to

$$\Gamma(\xi) = \frac{1}{\pi}\left(\frac{K}{U}\right)^2\frac{d\alpha}{dt}\frac{1}{\sqrt{\xi(1-\xi)}}\left[F(\xi)\log\left|\frac{\xi-1}{\xi}\right| + \sum_{m=0}^{N-1}\xi^m\sum_{j=1}^{N-m}a_{j+m}\frac{1}{j} - \sum_{j=1}^{N}a_j\frac{1}{j}\right] \tag{11}$$

The disturbance potential is

$$\phi = \frac{K\zeta^2}{2} + K\cos\alpha\int_0^\zeta \frac{(1+[1-\zeta^2]^{1/2})^{2-2n}}{\zeta^{1-2n}}d\zeta - \frac{1}{2}\int_\zeta^1 \Gamma(\zeta)d\zeta \tag{12}$$

Pressure at any point on the wetted surface of the wedge using unsteady Bernoulli's equation is

$$\begin{aligned} p = p_\infty + \frac{1}{2}\rho U_0^2\Bigg\{&\frac{1}{\pi I_1^2}\ddot\alpha\int_\zeta^1\Gamma^*(\zeta)d\zeta + \beta\dot\alpha\left[\frac{(I_1)'}{I_1^2}\zeta^2 + 2\sin\alpha\frac{I_2}{I_1} - 2\cos\alpha\left(\frac{I_2}{I_1}\right)' - \frac{2}{\pi}\Gamma^*(\zeta)\frac{I_3^2}{I_1}\zeta\right] - \dot\alpha^2\left[\frac{2}{\pi}\frac{(I_1)'}{I_1^3}\int_\zeta^1\Gamma^*(\zeta)d\zeta\right.\\ &\left. - \frac{1}{\pi I_1^2}\left(\int_\zeta^1\Gamma^*(\zeta)d\zeta\right)' + \frac{1}{\pi}\left(\frac{I_3}{I_1}\Gamma^*(\zeta)\right)^2 + \left(\frac{I_2}{I_1}\right)^2\right] + \dot\beta\left[\frac{\zeta^2}{I_1} + 2\cos\alpha\frac{I_2}{I_1}\right] + \beta^2(1 - I_3^2\zeta^2)\Bigg\} \end{aligned} \tag{13}$$

where

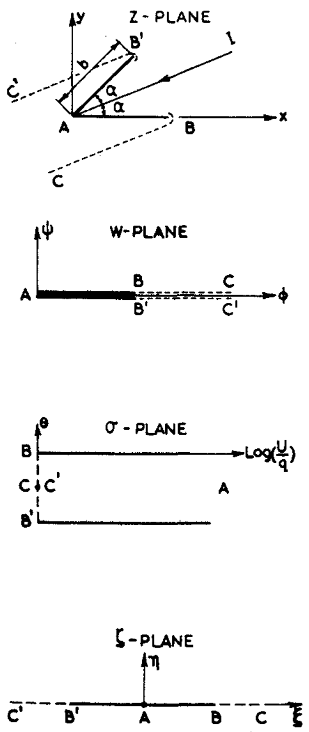

Fig. 2 Flow model.

$$\tau = U_0 t/b\,;\ \beta = U/U_0$$

$$I_3 = \frac{\zeta^{1-2n}}{(1+[1-\zeta^2]^{1/2})^{2-2n}}$$

$$I_1 = \int_0^1 d\zeta/I_3\,;\ I_2 = \int_0^\zeta d\zeta/I_3$$

$$(\dot{\ }) = \frac{d}{d\tau}(\ )\,;\ (\ )' = \frac{d}{d\alpha}(\ )$$

and $\Gamma^*(\zeta)$ is defined by the equation

$$\Gamma(\zeta) = \left(\frac{K}{U}\right)^2 \frac{1}{\pi}\frac{d\alpha}{dt}\Gamma^*(\zeta)$$

The Moment Equation

For dynamic equilibrium of the parachute system the total moment about any axis must be equal to zero. On equating the moment about an axis through the apex of the wedge, firstly due to aerodynamic pressure differential across the wedge surface, and secondly due to closing moment arising from rigging line tension, we have

$$\int_s psds - b\,\sin(\alpha+\delta)\,\sin\alpha/\cos\beta \int_s pds = 0 \qquad (14)$$

Substituting Eq. (13) in (14) and collecting like terms we obtain the following nonlinear second-order differential equation

$$C_1\ddot{\alpha} + C_2\dot{\alpha}^2 + C_3\beta\dot{\alpha} + C_4\dot{\beta} + C_5 = 0 \qquad (15)$$

where the coefficients C_1, C_2, C_3, C_4, and C_5 are all functions of wedge angle α and the ratio (L/b). The variation of these coefficients for $L/b = 2.0$ is shown in Fig. 3.

It can be seen that Eq. (15) is nonlinear in α and its time derivatives. Knowing the initial conditions [say $\alpha(0)$ and $\dot{\alpha}(0)$ are given] the equation is numerically evaluated by using the modified Range-Kutta method of Mersion.[6]

Conclusions

The aim of the study is to develop concrete ideas on how the parachute inflation process can be analyzed using analytical techniques. With the structural model shown in Fig. 1, the unsteady pressure distribution on the decelerating, inflating canopy surface is established. Then the moment equation yields the differential Eq. (15). This equation is solved for infinite mass case (i.e., $\dot{\beta} = 0$). From Fig. 4, it is clear the solution of the equation for α as a function of time, effectively replaces the filling-time notions inherent in the filling time theory. The equation basically describes a damped oscillation system in which two initial conditions [given $\alpha(0)$ and $\dot{\alpha}(0)$] will determine changes and palpatations in α as a function of time. The form of equation admits the over-inflation phenomenon followed by small damped oscillations in α until a steady inflated value $\bar{\alpha}$ is reached.

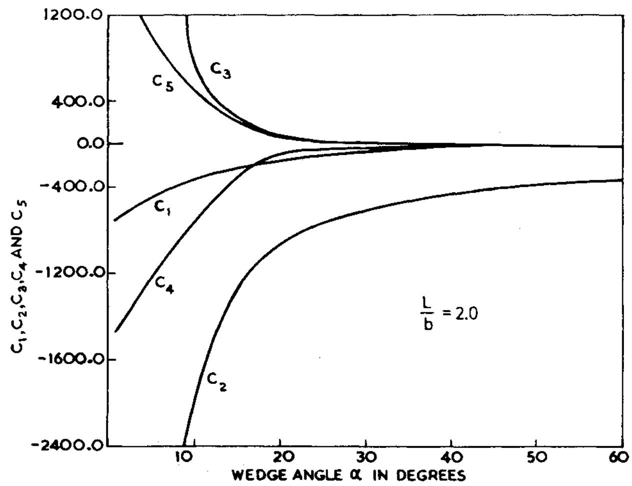

Fig. 3 Coefficients C_1, C_2, C_3, C_4, and C_5.

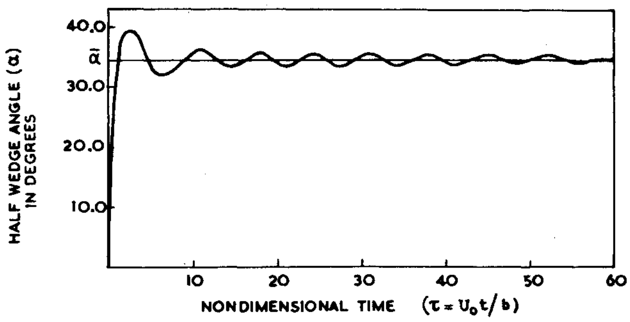

Fig. 4 Variation of wedge angle with time.

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Minimum Noise Climbout Trajectories of a VTOL Aircraft

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Nomenclature

x, h	= flight path coordinates
XP	= abscissa of observer point
R	= distance aircraft-observer
ϕ	= angle of radiation
PNL	= perceived noise level
EPNL	= effective preceived noise level

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